# Anoka-Hennepin Secondary Curriculum Unit Plan

| Department: | Mathematics | Course: | AP Calculus BC | Unit 2 Title: | Polynomial Approximations and Series | Grade Level(s): | 10-12 |
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| Assessed Trimester: | Trimester A | Pacing: | 12-18 days | Date Created: | 2/2/2010 | Last Revision Date: | 6/19/2014 |

**Course Understandings**: *Students will understand that:*

A. The meaning of limit represents function behavior.
B. The meaning of the derivative represents a rate of change and is a local linear approximation and should understand that derivatives can be used to solve a variety of problems.
C. The meaning of the definite integral is a limit of Riemann sums and as the net accumulation of change and will understand that you can use integrals to solve a variety of problems.
D. The relationship between the derivative and the definite integral as expressed in both parts of the Fundamental Theorem of Calculus.
E. You can model a written description of a physical situation with a function, a differential equation, or an integral.
F. You can use technology to help solve problems, experiment, interpret results, and support conclusions.
G. A polynomial series can be used to model functions and make approximations.
H. The analysis of planar curves can be done using the calculus of parametric, vector, and polar forms.

## DESIRED RESULTS (Stage 1) - WHAT WE WANT STUDENT TO KNOW AND BE ABLE TO DO?

### Established Goals

**Minnesota State/Local/College Board/Technology Standard(s) addressed:**

- **AP: IV. Polynomial Approximations and Series**
  a. **Concept of series**
    - A series is defined as a sequence of partial sums, and convergence is defined in terms of the limit of the sequence of partial sums. Technology can be used to explore convergence and divergence.
  b. **Series of constants**
    - Motivating examples, including decimal expansion
    - Geometric series with applications
    - The harmonic series
    - Alternating series with error bound
    - Terms of series as areas of rectangles and their relationship to improper integrals, including the integral test and its use in testing the convergence of *p*-series
    - The ratio test for convergence and divergence
    - Comparing series to test for convergence or divergence
  c. **Taylor series**
    - Taylor polynomial approximation with graphical demonstration of convergence (for example, viewing graphs of various Taylor polynomials of the sine function approximating the sine curve)
    - Maclaurin series and the general Taylor series centered at $x = a$
    - Maclaurin series for the functions $e^x$, $\sin x$, $\cos x$, and $\frac{1}{1-x}$
    - Formal manipulation of Taylor series and shortcuts to computing Taylor series, including substitution, differentiation, antidifferentiation, and the formation of new series from known series
    - Functions defined by power series
    - Radius and interval of convergence of power series
    - Lagrange error bound for Taylor polynomials

### Transfer

**Students will be able to independently use their learning to: (product, high order reasoning)**

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## Meaning

| Unit Understanding(s): | Essential Question(s): |
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| **Students will understand that:**<br>● The concept of a series, including geometric, Taylor, and Maclaurin series.<br>● Several tests for convergence and divergence. | **Students will keep considering:**<br>● Can any function be represented by a polynomial?<br>● What does it mean for a series to converge or diverge?<br>● How many tests for convergence and divergence are there? |

## Acquisition

| Knowledge - Students will: | Skills - Students will: |
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| ● Convergence tests<br>● Taylor, Maclaurin, and Power Series<br>● Error<br>● Interval of Convergence<br>● Radius of Convergence<br>● Infinite Series<br>● Sum of a geometric series<br>**Reasoning - Students will:**<br>● Distinguish which convergence test is appropriate<br>● Verify that Taylor, Maclaurin, and Power series models a certain function within an interval of convergence<br>● Determine if error is within a certain bound<br>● Analyze circumstances for which an infinite series will have a finite sum<br>● Evaluate approximation accuracy<br>● Compare/Contrast the relationship between functions and series using integration and differentiation | ● BC2-1: Determine convergence/divergence of a series using the appropriate test<br>● BC2-2: Create Taylor, Maclaurin, and Power series to model functions<br>● BC2-3: Calculate an error in using a polynomial to approximate a function<br>● BC2-4: Find the sum of an infinite series<br>● BC2-5: Find an interval of convergence/radius of convergence<br>● BC2-6: Integrate/Differentiate an infinite series |

| Common Misunderstandings | Essential new vocabulary |
|---|---|
| ● Students have trouble calculating the coefficients of Taylor Polynomials (failure to divide by the factorial)<br>● Students confuse a Taylor Polynomial centered at 0 and not at 0<br>● Students have trouble with calculating actual error vs. using error estimation techniques | ● Convergence<br>● Divergence<br>● Harmonic Series<br>● LaGrange Error Bound<br>● Maclaurin<br>● P-Series<br>● Truncation error<br>● Taylor Series |